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A.R.D.E. MEMORANDUM (B) 63/58

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A sampling method of determining chances of hit and kill under quasi-battle conditions

FOR RETENTION

S. A. Beach.

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ARMAMENT RESEARCH AND DEVELOPMENT ESTABLISHMENT

A.R.D.E. MEMORANDUM (B)63/58

A sampling method of determining chances of hit and kill under quasi-battle conditions

S.A. Beach (B5)

Summary

This paper details a sampling method by means of which realistic chances of hit and kill may be quickly and conveniently calculated.

The advantages of this technique are:-

(a) that it does not separate the evaluation of chance of hit from chance of kill and thus calculates the averaged chance of kill, $\bar{P}_k$, as an averaged product $\overline{P_h \times P_{kh}}$;

(b) that the target silhouette can be treated realistically for both chance of hit and chance of kill calculations, and that the same assumptions will apply in each case;

(c) that it is no longer necessary in chance of kill calculations to restrict the treatment, as is usual in methods used to date, by the non-realistic assumption that hits occur in the target area at random;

(d) that in the method outlined in this paper there is no longer any need to restrict chance of kill assessments only to single shot weapons.

Approved for issue:

J.W. Maccoll, Principal Superintendent "B" Division

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INTRODUCTION

1. In this paper a sampling method for the calculation of chances of hit and kill will be presented. In the first section the background of the study will be discussed and the advantages of the sampling method of calculation detailed. In the second section the new technique is developed for chance of hit and kill calculations and an example of its use is given.

2. The assessment of tank and anti-tank weapons is specifically mentioned throughout this report because it is in this field that the need for such a method has been most recently apparent. The methods outlined are, however, of general application once the component errors entering into such calculations are known.

BACKGROUND

3. The chance that a round fired from a gun will kill a tank depends upon

(a) the chance that the tank is hit (P_h)

(b) the chance that this hit will kill the tank (P_{kh})

4. It has been usual in weapon assessments to calculate both these probabilities as weighted averages over all aspects of attack separately, and then to obtain an estimate of the average overall chance of kill by multiplying them together according to the formula,

$$\text{(i)}\quad \overline{P_k} = \overline{P_h} \times \overline{P_{kh}}$$

where $\overline{P}_k$ is the averaged chance of kill, $\overline{P}_h$ is the averaged chance of hit and $\overline{P}_{kh}$ is the averaged chance of kill given a hit. The quantity $\overline{P_k}$ as calculated by the scheme represented by equation (i) may differ, to some extent, from the true chance of kill which may be represented by the formula,

$$\text{(ii)}\quad \overline{P_k} = \overline{P_h \times P_{kh}}$$

where the quantity $\overline{P_h \times P_{kh}}$ is the averaged combined chance of kill of all possible shots from the equipment against the true target tank viewed from all aspects in turn. In other words, equation (i), which represents formally methods previously used for the calculation of $\overline{P}_k$, assumes that there is no correlation between the chances of hitting and killing the tank.

5. Other objections to the use of the scheme outlined in equation (i) are that $\overline{P}_h$ is normally computed for a stylized target representation which may depart in practice to some unknown extent from reality, and that $\overline{P}_{kh}$ is normally calculated from the assumption that hits occur in the target area at random.

6. In the next section a brief introduction will be given to sampling methods of calculation; then factors that affect chance of hit will be described, and this will be followed by a discussion of those factors that affect chance of kill. On the basis of these arguments it will be shewn how the sampling method given has been developed.

SAMPLING METHOD

General

7. The sampling, or Monte Carlo, technique* (1) (2) is used in the treatment of deterministic or probabilistic problems which are either difficult or impossible of solution by the more usual methods of mathematical analysis.

8. In chance of hit and kill assessments by this method an artificially produced population of shot strikes in the target plane is first constructed using the component errors determined for the weapon and ammunition under consideration. The history of each shot strike in the target plane is then recorded and the damage sustained by the target tank from each hit is assessed and also recorded.** Thus, a shot might hit and kill the tank, or it might hit but not kill, or it might miss the tank. The process is continued, each aspect of the target being taken in turn, until the parameters of chance of hit and kill are considered sufficiently accurately determined. In effect, the method aims to reproduce as exactly as possible in simulation the actual firing of the weapon at the tank.

Chance of hit

9. The first round chance of a shot hitting a tank under battle conditions has been shown in previous work (3) to depend on three main classes of errors, which have been tabulated according to their ultimate effect upon the points of strike of all possible shots fired on all occasions from a series of weapons of the same type. These are:-

(a) permanent biases

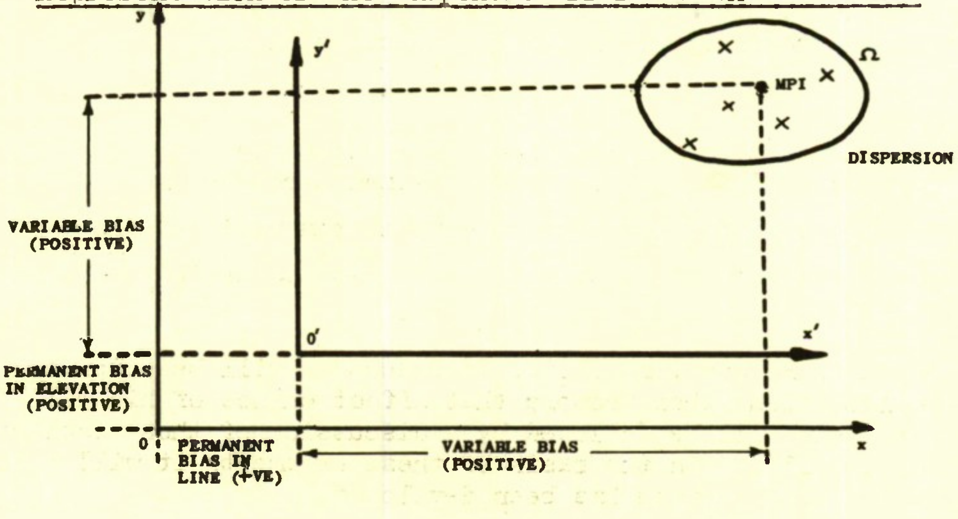
(b) variable biases

(c) random errors, or dispersions

The final effect of these factors may be conveniently represented by the following diagram (Figure 1) where the resultant permanent bias is a shift of origin, (here shown positive for line and elevation) from 0 to 0' ; the resultant variable bias determines where in the plane the MPI is likely to be on a particular occasion from this new origin 0', and the resultant dispersion defines an area Ω about the MPI in which all or most of the shots fired on a particular occasion lie.

Figure 1

Representation of the component errors in chance of hit



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* The numbers in brackets throughout this paper refer to the list of references at the end.

** The idea of what constitutes a "kill" differs, naturally, from target to target; the criteria used for tank targets are discussed in references (4) (5) at the end of this report.

10. It is clear that a sampling process for the calculation of chance of hit must follow the scheme outlined in Figure 1. In other words, it is necessary to relate the permanent bias to a shift of origin for a particular weapon, ammunition and range, and to keep this constant throughout the sampling process. The position of the MPI for each burst must then be discovered by sampling from the bivariate probability surface within which the MPI must lie; the position of each shot within the burst area, as previously defined, about the MPI must then be found by a second sampling process for each separate shot of the burst from the bivariate probability surface defined by the resultant random dispersions about the MPI. It should be noted that both salvo and burst firing are automatically covered by such a sampling method, whereas their solution by earlier methods was both difficult and lengthy. By this process therefore a sample of points of strike in the target plane is obtained which has the component errors appropriate to the gun, range and ammunition being considered.

Chance that a hit kills

11. In chance of kill calculations it is first necessary to specify the dimensions of the target in as great a detail as possible, including the thickness and distribution of armour plating, if this is present, and the positions of the principal vulnerable components. The aspects of attack must also be taken into account as must the performance of the particular shot concerned on target impact.

12. A great deal of work has been done in this field and many assessments of vulnerability of particular targets to given weapons undertaken. The method followed against tank targets (4) (5) may be taken as typical of the basic approach and involves essentially the calculation of vulnerable areas in relation to the whole area subtended at a particular aspect, thickness and obliquity of the armour plating being taken into account. All aspects of attack are considered at, say, 15° intervals, suitable weighting is also given to the directional probability variation (DPV) which denotes the chance of attack from various aspects.

Determination of chance of kill

13. The method employed by the Monte Carlo technique is to superimpose on the target, in effect, a series of shots (produced in the manner discussed in para 10), to record the result of any hits (it will be assumed that the performance of shot is known) and to repeat for different aspects of attack. The sampling procedure thus also has the advantage that no assumption need be made that hits occur at random over the target surface.

14. The procedure outlined in the above paragraphs is thus completed in one logical operation, symbolized by equation (ii) of para 4; the calculation of chance of hit is not separated from the calculation of chance of kill. In the next section the practical use of the sampling method of calculation of chance of kill will be described.

USE OF THE TECHNIQUE

Practical method of calculation

15. The first difficulty encountered in the practical calculation of chance of kill by the Monte Carlo method is that of the accurate, quick and convenient generation of the artificial population of shell strikes in the target plane. Ideally these should be given in the form of coordinates of strike referred to an origin assumed to be the point of aim suitable to the target form and type. This process may be done by hand; in which use is made of a table of random Normal deviates, each such deviate being

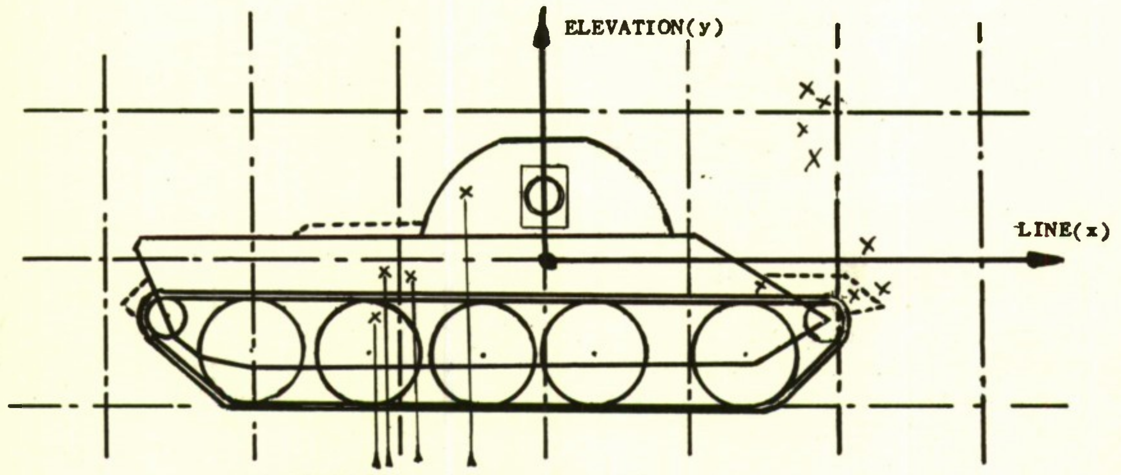
paired with the error standard-deviation found to apply in line and elevation. Such a method is extremely slow and tedious as care must be taken at all stages of the calculation. A programme was therefore constructed for use in the high speed digital computer AMOS* that automatically produces samples, of any required size, of shot strikes in the target plane.

16. It is then necessary to construct accurate scale drawings of possible tank targets from all angular aspects by increments of, say, 15°. It has been found convenient to express all errors and target tank dimensions in the angular measure of mils subtended at the gun for the particular range at which chance of kill is being computed.

17. A sample of shot strikes, conveniently in coordinates of line and elevation similarly measured in mils at the same range, can then be plotted on to semi-transparent graph paper already laid and centred on the point of aim of the target. The effect of each shot is then recorded. This process is typified in Figure 2 and Table 1 below. The overall chance of kill, or hit, over all aspects, can then be easily calculated as an Arithmetic mean, weighted by the probability of attack at each aspect (Whittaker's DPV) within any required degree of accuracy. The precision of the result will, in fact, vary inversely as the square-root of the number of bursts considered.

Figure 2

Graphical plot of shots in the target-plane



Example

Table 1

A typical tabulation of shot strikes in the target plane

Aspect	Burst	Lethality of each round 1	2	3	4	Result
	1	0	2/3	M	2/3	89%
	2	2/3	2/3	M	M	89%
	3	2/3	2/3	2/3	2/3	99%
90°	4	0	0	0	0	0
	5	2/3	0	M	0	67%
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	200	0	0	2/3	M	67%
90°	Total					47.07%

Where 0 = a hit but not a kill; 2/3 = a penetration, assessed to be a 2/3rds kill; M = a miss.

* This is the Ferranti Mk 1* electronic computer currently in use in ARDE; the programme together with all relevant notes has been issued for internal distribution within ARDE, a flow diagram is shown in Figure 4.

18. As an example, purely for the purposes of illustration, of this method in practice and to show the comparative ease with which burst firing may be encompassed by the sampling method, a fictitious rapid-firing anti-tank weapon with arbitrarily chosen biases, variable biases, and random dispersions, is assessed for chance of kill in Appendix A. Table 1 above, may then be considered as giving some of the results of this assessment. In this table the tank is inclined at an angle of 90° to the weapon which is firing; there are therefore six other aspects, all similarly tabulated, for the angles between 0° and 75° inclusive. The percentage mean chance of kill for each aspect is then multipled by the weighting factor of the chance that the tank will be attacked from that particular angle (see also para 12) and a grand overall mean taken of the chance of kill.

SUMMARY AND DISCUSSION

19. A sampling method has been detailed in the foregoing paragraphs by means of which realistic chances of hit and kill may be quickly and conveniently calculated.

20. The advantages of this technique are:-

(a) that it does not separate the evaluation of chance of hit from chance of kill and thus calculated the averaged chance of kill, $\overline{P_k}$, as an averaged product, $\overline{P_h \times P_{kh}}$, instead of as the product of two separately calculated averages, $\overline{P_h} \times \overline{P_{kh}}$;

(b) that the target silhouette can be treated realistically for both chance of hit and chance of kill calculations, and that the same assumptions will apply in each case;

(c) that it is no longer necessary in chance of kill calculations to restrict the treatment by the non-realistic assumption that hits occur in the target area at random;

(d) that in the method now outlined there is no longer any need to restrict chance of kill assessments only to single shot weapons; burst and salvo firing assessments previously difficult of solution follow quite naturally by normal use of the Monte Carlo method.

APPENDIX A

Assessment of chance of kill

1. In this section the method outlined in the main body of this paper is followed in the assessment of a chance of kill for a fictitious quickfiring anti-tank gun. The rapid-firing example has been chosen to illustrate the comparative ease with which chance of kill for consecutive burst firing may be dealt with; a similar reasoning holds true for the other types of firing that have proved difficult of solution by earlier methods such as that of salvo firing.

2. It will be assumed that the chance of kill is to be calculated against a possible future enemy tank at 1200 yards range.

3. Values for the component errors are given in Table 1 below;-

Table 1

Component errors of fictitious quick-firing A tk gun

ERROR	Permanent bias	Variable bias	Dispersion
Elevation (y)	0	0.800 /m sd	0.700 /m sd
Line (x)	R0.035 /m	0.700 /m sd	0.600 /m sd

These errors are cognizant with a light barrelled portable gun firing 90 mm, HEAT FS projectiles in bursts of four at a rate of, say, fifteen rounds per minute, with a ballistic error of the order of 0.6 /m and 0.5 /m in elevation and line respectively; aim-wander correlation will, in this instance, be ignored.

4. The first sixteen bursts produced by the electronic computor programme are shown in Figure 5 this gives the coordinates, in mils, of the points of strike of the shot. In each line, the x-coord is given first and the y-coord directly underneath. The starting values are printed at the head of the data printing, giving first the dispersion values in line and elevation and then the variable and permanent biases. The decimal printed on the extreme left is the burst number printed in this way to save using an additional sub-routine.

5. The coordinates of strike are then transferred to a semi-transparent graph paper where the ruling is considered scaled in mils and tenths of a mil and the paper placed and centred on a scale-drawing of a possible enemy tank, at a particular aspect of, say, 0° in the first instance. If the strike is a hit the effect can then be easily determined from the thickness of armour at the point of contact and the relative positions of vulnerable components. The first sixteen bursts are shown plotted onto a typical tank target at 0° aspect of attack in Figure 3; for greater clarity the graph-paper ruling is only lightly indicated in this diagram.

6. The results can then be tabulated in the manner shown in Table 1 of the main body of this report; the actual results of the sixteen bursts of Figure 5 are given below in Table 2 of this appendix. This process is repeated for each aspect, and the results weighted by whatever DPV, as for example that due to Whittaker (4)(5), is considered suitable. The chance of kill is then computed as an Arithmetic mean of these weighted results.

Table 2

* Tabulation of shot strikes in target plane

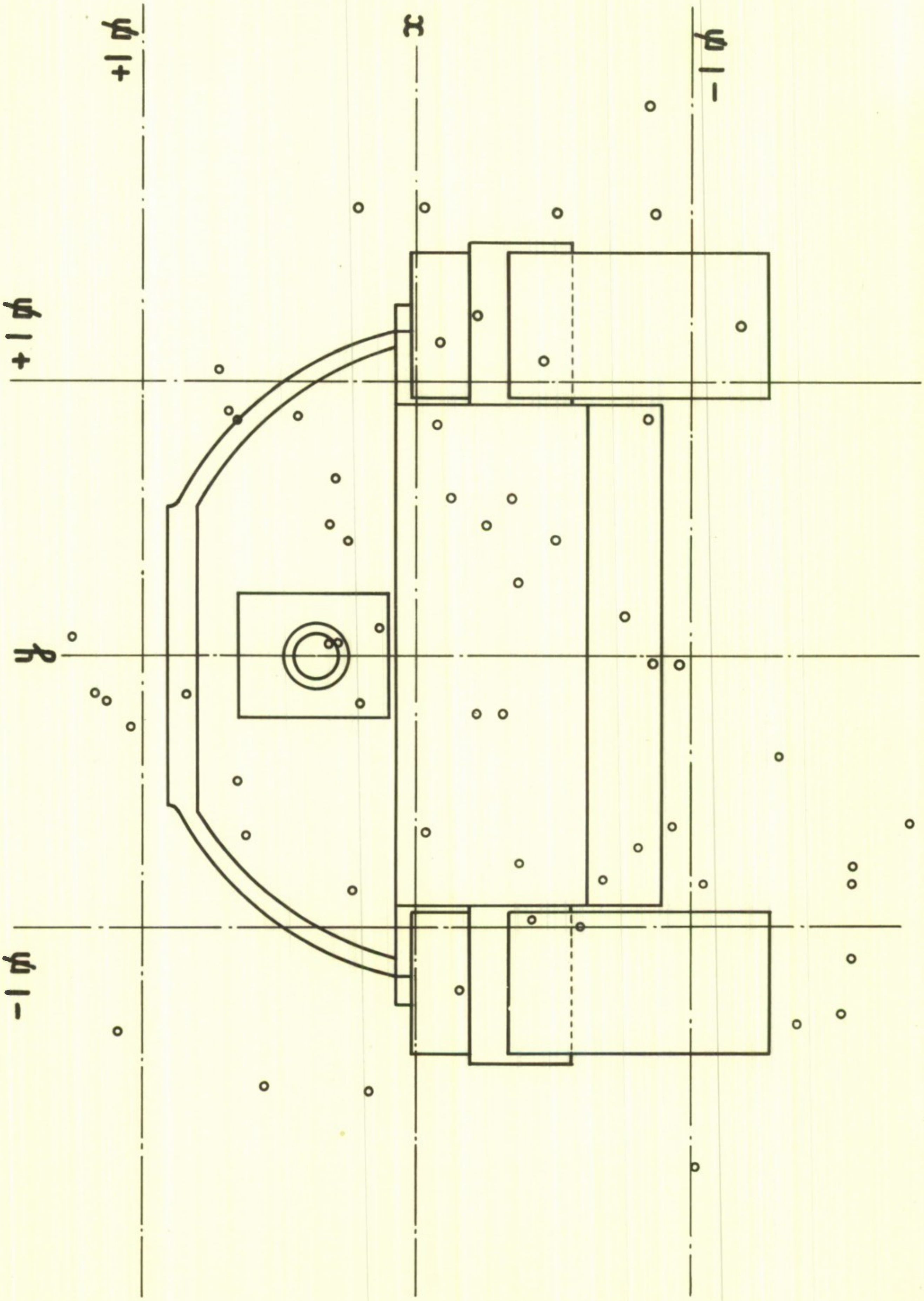
Aspect	Burst	Lethality of each round				Result
		1	2	3	4	
	1	M	2/3	2/3	2/3	96%
	2	2/3	0	M	M	67%
	3	0	M	M	0	0
	4	M	0	2/3	0	67%
	5	M	M	M	M	0
	6	2/3	M	M	0	67%
	7	2/3	2/3	2/3	M	96%
	8	2/3	0	2/3	2/3	96%
	9	2/3	2/3	2/3	2/3	99%
	10	M	M	M	M	0
0°	11	M	M	M	2/3	67%
	12	M	2/3	2/3	2/3	96%
	13	2/3	2/3	2/3	M	96%
	14	M	M	2/3	2/3	89%
	15	M	2/3	0	0	67%
	16	M	M	M	M	0
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	200					
0°	Total					

* The Coords of these shots are shewn in Figure 5: the same points of strike are plotted onto a tank silhouette at 0° in Figure 6.

REFERENCES

(1)	Meyer, H.A. (Editor)	Symposium on Monte-Carlo methods. NY USA, Wiley & Sons Inc. 1956.
(2)	Householder, A.S. Forsythe, G.E. and Germond, H.H. (Editors)	Monte Carlo Method, USA, Nat. Bureau of Standards, AM-12, June 1951.
(3)	ARDE Report (B) 6/58	The classification of errors involved in the determination of chances of hit and the evaluation of chance of hit with Centurion under quasi-battle conditions.
(4)	AORG Report 8/54	The conventional tank: A study of performance and cost and the selection of an optimum weight.
(5)	ARDE Memorandum (B) 30/57.	The vulnerability of tanks to attack by hollow-charge projectiles

FIG. 3.



POINTS OF STRIKE PLOTTED ONTO A TANK SILHOUETTE AT 0°

FIG. 4

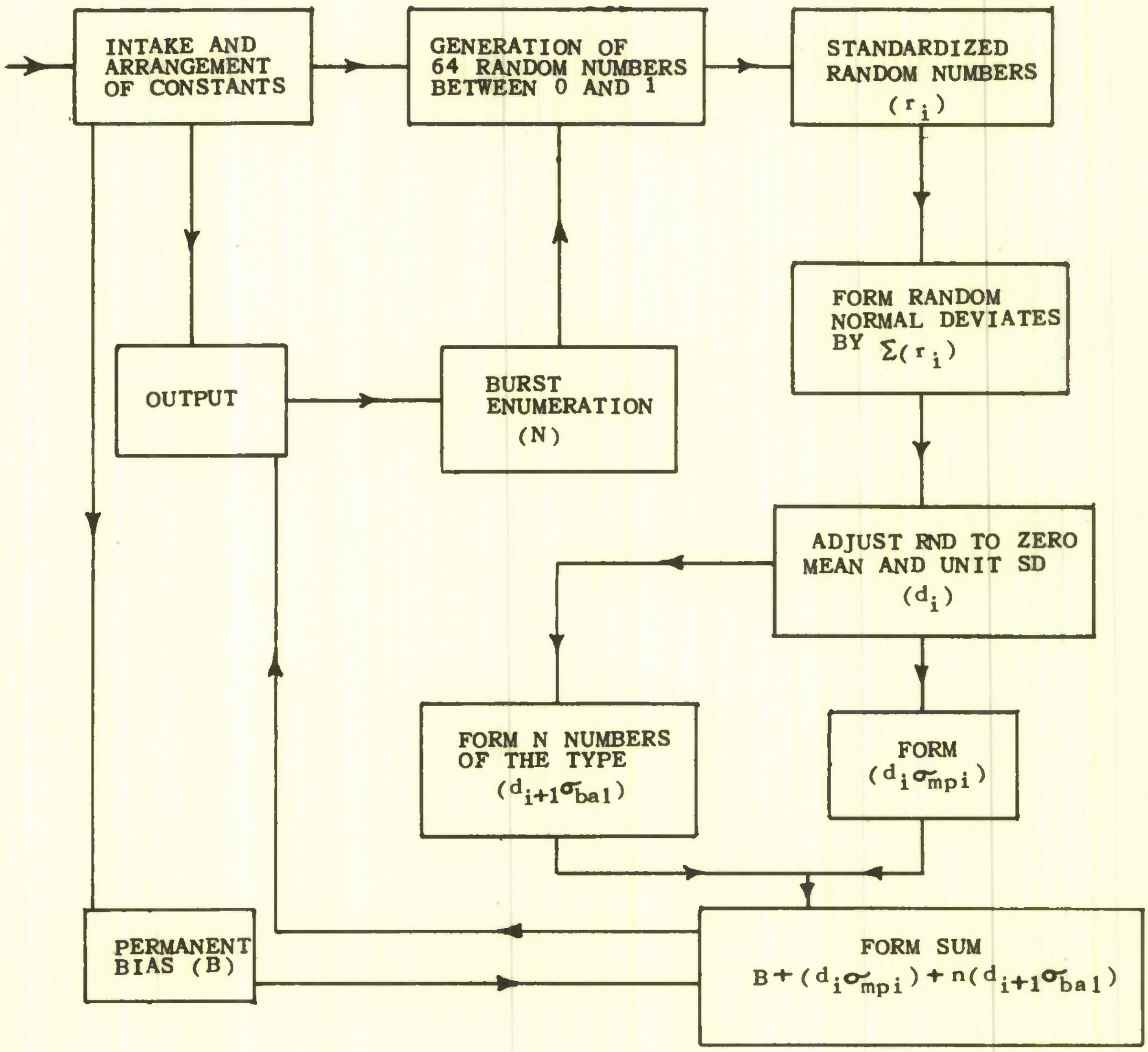


FIG. 4 FLOW DIAGRAM ILLUSTRATING COMPUTER PROGRAMME

Figure 5

Example of output showing coordinates of points of strike in mils at 1200 yards

0

RDS/BURST=4 BURSTS=200

0.6000	0.7000	0.8000	0.7000	0.0350	0.0000
0.001	-1.38	-0.78	-0.02	-0.23	
0.001	-1.59	-0.37	-0.89	-0.22	
0.002	-0.87	-1.24	-0.63	-1.91	
0.002	0.25	-0.17	-0.93	-1.04	
0.003	1.09	1.61	-0.04	1.48	
0.003	-0.48	-0.90	-0.97	-1.21	
0.004	-1.56	-0.98	-0.84	-1.02	
0.004	0.57	-0.41	-0.67	-0.60	
0.005	-0.41	0.06	-0.15	-1.11	
0.005	2.51	1.27	1.18	2.38	
0.006	0.89	0.90	1.61	1.14	
0.006	0.43	0.68	1.73	-0.17	
0.007	-0.66	-0.46	-0.18	-0.40	
0.007	-0.04	0.67	0.20	-1.35	
0.008	0.65	0.87	0.04	0.58	
0.008	0.30	0.66	0.29	-0.13	
0.009	0.43	0.89	0.15	0.86	
0.009	0.25	-0.89	-0.77	-0.07	
0.010	1.64	1.05	1.65	2.03	
0.010	-0.54	0.74	-0.04	-0.88	
0.011	-0.79	-0.86	-1.62	-0.71	
0.011	-1.61	-1.61	0.19	-0.83	
0.012	-1.14	-0.21	0.58	0.26	
0.012	-1.63	-0.32	-0.36	-0.39	
0.013	0.04	0.48	0.10	1.65	
0.013	0.32	0.31	0.13	0.21	
0.014	-0.17	-1.38	-0.69	0.48	
0.014	1.16	1.10	0.64	-0.26	
0.015	-0.26	0.41	1.25	-0.16	
0.015	1.06	-0.58	-0.23	0.85	
0.016	-1.38	-0.85	-0.62	-0.34	
0.016	-1.40	-1.06	-1.81	-2.09	

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